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A Method to Determine Reasonable Correcting Planes for Balancing of Flexible Rotors

Ri Yong Ho[*], *Kang Il Yong*

School of Science and Technology, Kim Chaek University of Technology, Pyongyang, DPRK

[*]Corresponding author: Email: ryh70824@star–co. net. kp

Summary

Author has been developed a new method which can improve preciseness of the influence coefficient method. By Component Mode Synthesis (CMS), this method determines reasonable correcting planes that transfer unbalances distributed along the length of a rotor into the set of equivalent concentrated unbalances at rotating speeds below service speed. Numerical Simulation results show that this method can equivalent significantly unbalance distribution of flexible rotors to a set of concentrated unbalances.

Keywords: flexible rotor; balancing; CMS

1. Introduction

The influence coefficient method and modal balancing have been developed and used mainly for balancing of flexible rotors.[1~4] The influence coefficient method is based on the assumption that unbalance response of a rotor is proportional to unbalances. The influence coefficient method determines influence coefficient matrix of a rotor experimentally by known trial masses and calculates a set of discrete correction masses that will minimize whirl responses. When rotating speed of a flexible rotor is changed, the influence coefficient is changed strictly because the dynamical characteristic is changed according to speeds. Therefore, the influence coefficient matrix has to be determined again experimentally. This method is extended by applying the least–squares method and other optimization methods.[5~7] While the modal balancing method is mode–by–mode balancing method and uses graduated procedures. [8] In this method, sets of orthogonal masses are calculated and applied to balance each mode component without affecting the previous balance of lower modes. In the method, a rotor has to be operated at around of critical speeds to balance all modal components below the service speed, so it is dangerous. In this method, balancing of a flexible rotor requires a large number of test runs to measure vibration data for multi correcting planes at different speeds. Author proposes a new method using CMS (Component Mode Synthesis) to determine reasonable correcting planes. The main goal of this method is to determine reasonable correcting planes that can equivalent unbalance distribution of a flexible rotor into a set of concentrated unbalances at all rotating speeds below service speed by CMS.

2. Theoretical backgrounds

In general, a rotor–bearing system is composed with shaft, discrete discs and discrete bearings. For analyzing unbalance response of a flexible rotor, governing equation of the system is written as follow by using FE method.

$$M\ddot{X} + C\dot{X} + KX = f \tag{1}$$

In case that a rotor system has proportional damping and gyroscope effect is neglected, steady unbalance response of a flexible rotor operating at a speed Ω is

$$X_i(\Omega) = \sum_{j=1}^{N}\sum_{r=1}^{m} \frac{\Omega^2 \phi_{i,r}\phi_{j,r}}{\omega_r^2 - \Omega^2 + j2\varsigma_r\omega_r\Omega} U_j \tag{2}$$

where U_i is unbalance of j DOF, $\varphi_{i,r}$ and $\varphi_{j,r}$ are i, j th DOF element of rth mode, N is number of DOFs of FE model and m is the number of modes considered. Eq. (2) is rewritten as follow.

$$\{X(\Omega)\} = \begin{bmatrix} \phi_{1,1} & \phi_{1,2} & \cdots & \phi_{1,m} \\ \phi_{2,1} & \phi_{2,2} & \cdots & \phi_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{N,1} & \phi_{N,2} & \cdots & \phi_{N,m} \end{bmatrix} \begin{bmatrix} \alpha_1(\Omega) & 0 & \cdots & 0 \\ 0 & \alpha_2(\Omega) & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & \cdots & \alpha_N(\Omega) \end{bmatrix} \begin{bmatrix} \phi_{1,1} & \phi_{2,1} & \cdots & \phi_{m,1} \\ \phi_{1,2} & \phi_{2,2} & \cdots & \phi_{m,2} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{1,N} & \phi_{2,N} & \cdots & \phi_{m,N} \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ \vdots \\ U_m \end{Bmatrix}$$

$$\alpha_r(\Omega) = \frac{\Omega^2}{\omega_r^2 - \Omega^2 + j2\varsigma_r\omega_r\Omega} \tag{3}$$

$\alpha_r(\Omega)$ is defined as the dynamical amplification coefficient of rth critical frequency at a rotating speed Ω.

$$\{\xi(\Omega)\} = \begin{bmatrix} \alpha_1(\Omega) & 0 & \cdots & 0 \\ 0 & \alpha_2(\Omega) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_N(\Omega) \end{bmatrix} \times \begin{bmatrix} \phi_{1,1} & \phi_{2,1} & \cdots & \phi_{m,1} \\ \phi_{1,2} & \phi_{2,2} & \cdots & \phi_{m,2} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{1,N} & \phi_{2,N} & \cdots & \phi_{m,N} \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ \vdots \\ U_m \end{Bmatrix} \tag{4}$$

$$\{X(\Omega)\} = [\phi][\alpha(\Omega)][\phi]^T\{U\} = [\phi]\{\xi(\Omega)\} \quad (5)$$

A rotor–bearing system is supposed one component and divided to inner DOFs (i DOFs) and junction DOFs (j DOFs). Steady unbalance responses at i DOFs of the component are defined as follows at service speed Ω, under the supposition of proportional damping.

$$\{X_i(\Omega)\} = [\phi_c]\{X_j(\Omega)\} + [\phi_i^n]\{\xi_n(\Omega)\} \quad (6)$$

Therefore, by using Eq. (4), it is possible to write as follows.

$$\{\xi_n(\Omega)\} = [\alpha_n(\Omega)][\phi_i^n]^T\{U^n\} \quad (7)$$

$$\alpha_{n,\,r}(\Omega) = \frac{\Omega^2}{\omega_{n,\,r}^2 - \Omega^2 + j2\varsigma_{n,\,r}\omega_{n,\,r}\Omega} \quad (8)$$

where $\omega_{n,\,r}$ is r th critical speed of the component.

By determining j DOFs reasonably, it is possible that the condition $\omega_{n,\,1} \gg \Omega$ is satisfied and consequently the condition $\alpha_{n,\,1} \approx 0$ is also satisfied. In case that the above conditions are satisfied, because the condition $\Omega \ll \omega_{n,\,1} < \omega_{n,\,2} \cdots \omega_{n,\,N}$ is also satisfied, it is possible to write as follows.

$$[\alpha_n(\Omega)] \approx [0] \quad (9)$$

Therefore, Eq. (6) can be rewritten as follow in this case.

$$\{X_i(\Omega)\} = [\phi_c]\{X_i(\Omega)\} \quad (10)$$

Eq. (10) means that steady unbalance responses at i DOFs are related only to j DOFs, if j DOFs are determined to satisfy Eq. (9) on the component. This also means that if steady unbalance responses at j DOFs are zero, steady unbalance responses are zeros at all DOFs of the flexible rotor bearing system. While it is possible to write for the flexible rotor bearing system as follows.

$$\{X_j(\Omega)\} = [\phi_j]\{\xi(\Omega)\} = [\phi_j][\alpha(\Omega)][\phi_j]^T[I[\phi_c]^T]\{U\} \quad (11)$$

$$\{X_i(\Omega)\} = [\phi_i]\{\xi(\Omega)\} = [\phi_i][\alpha(\Omega)][\phi_i]^T[I[\phi_c]^T]\{U\} \quad (12)$$

From Eq. (11) and (12), unbalance distribution $\{U\}$ is equivalent to a set of unbalances concentrated at j DOFs.

$$\{U_C\} = [I[\phi_c]^T]\begin{Bmatrix} U_j \\ U_i \end{Bmatrix} \quad (13)$$

$$\{X(\Omega)\} = [\phi][\alpha(\Omega)][\phi_j]^T\{U_C\} \quad (14)$$

Eq. (13) means that unbalance distribution of the rotor shaft can be transferred to a set of concentrated unbalances at j DOFs, if j DOFs are determined reasonably to satisfy Eq. (9) on the rotor. Also, in case that Eq. (9) is satisfied, Eq. (10), (14) are satisfied at a rotating speed $\omega(\omega<\Omega)$ consequently. Then, steady unbalance response of a flexible rotor is as follows.

$$\{X_i(\omega)\} = [\phi_c]\{X_j(\omega)\} \quad (15)$$

$$\{X(\omega)\} = [\phi][\alpha(\omega)][\phi_j]^T\{U_C\} \quad (16)$$

Therefore, j DOFs are reasonable measuring planes and furthermore, correcting planes at a rotating speed $\omega(\omega<\Omega)$.

3. Numerical simulation

The method proposed in this paper is tested numerically by simulating harmonic response of a rotor–bearing system. Simulation is done for the rotor–bearing system. The rotor–bearing system is consisted of a steel shaft and two rolling element bearings, while the shaft is supported by bearings at two ends. Diameter of the shaft is 45mm, length is 1 200mm, equivalent support stiffness of the two end bearings is 1e9N/m and proportional damping is supposed. Balancing of the rotor has been carried out numerically by using MATLAB (Ver. 9). FE model of the rotor–bearing system was constructed and the harmonic response behavior was simulated for mass unbalance distribution imposed to nodes of the FE model.

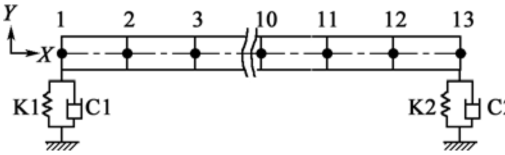

Figure 1. FE model of the rotor bearing system

The rotor shaft was meshed to twelve 2D beam elements and the support bearings were modeled by spring–damper elements.

By FE model, critical speeds of the rotor bearing system were obtained. As result, the first, second and third natural frequencies are 63.7, 256.9 and 586.2Hz. Therefore, service speed of the rotor is between the second and third natural frequency. For three j DOFs, when translational DOFs of node 4, 7 and 10 in Y direction were constrained, the first critical frequency is the maximum value 1 100.3Hz and the dynamical amplification coefficient at service speed was 0.05 when damping was neglected. Under the supposition that Eq. (9) was satisfied for the dynamical amplification coefficient, the balancing and measuring planes were defined at positions of node 4, 7 and 10. Therefore, by the method proposed, nodes 4, 7 and 10 are determined to the positions of correcting planes in order to satisfy Eq. (9) at speeds below 300Hz. By the influence coefficient method, magnitude and phase of the unbalances at the correcting planes were obtained and characteristics of them according to rotating speed are exhibited below. Properties of the FE model

Table 1. Properties of the FE model

Element no	Element nodes	Element length, m
1	1, 2	0.1
2	2, 3	0.1
3	3, 4	0.1
4	4, 5	0.1
5	5, 6	0.1
6	6, 7	0.1
7	7, 8	0.1
8	8, 9	0.1
9	9, 10	0.1
10	10, 11	0.1
11	11, 12	0.1
12	12, 13	0.1

Magnitude and phase of the unbalances at the correcting planes are obtained by the influence coefficient method and characteristics of them according to rotating speed are exhibited below. As shown Fig. 2 and 3, magnitude and phase of unbalances at the correcting planes were almost invariable according to speeds and, as the speed is increased, changed sensitively. It is because that the dynamical amplification coefficient was increased according to speeds.

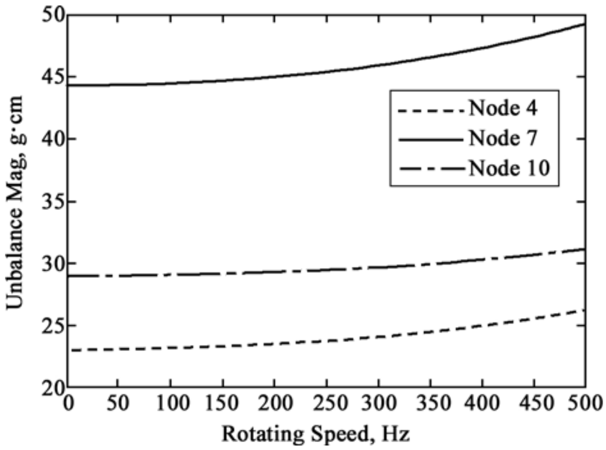

Figure 2. Magnitude of the equivalent concentrated unbalances according to rotating speed at node 4, 7 and 10.

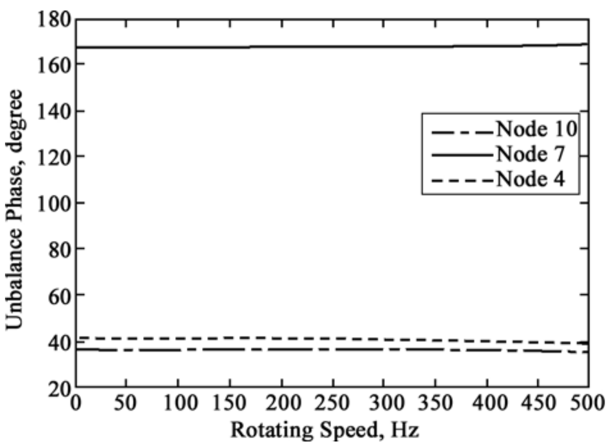

Figure 3. Phase of the equivalent concentrated unbalances according to rotating speed at node 4, 7 and 10.

From the simulation results, it is obvious that this method can determine reasonable correcting planes which transfer unbalance distribution according to the length of the shaft into a set of concentrated unbalances. The set of concentrated unbalance is corresponded to intrinsic unbalance distribution of the shaft and therefore, independent of speed under the certain condition.

4. Conclusions

This paper presents a new effective method to determine reasonable correcting planes for balancing of flexible rotors by using CMS. The main principle of this method is that, at all rotating speeds below goal speed, unbalance distribution of a flexible rotor can be transformed into a set of equivalent concentrated unbalances, which is independent of speed when balancing planes are determined reasonably.

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