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NN–based Identification and Adaptive Feedback Linearization Control for Nonlinear Systems

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Summary

We proposed a new approach of control system design using single–hidden–layer neural networks (NN) for the unknown nonlinear dynamic system. In a number of methods, recently applied well to nonlinear control system using NN, the NN is directly trained by error signal between system output and reference signal, and the boundedness of both the tracking error and learning error are guaranteed by Lyapunov's stability theory. By generalizing this principle we suggested the normal learning system in which unknown nonlinear dynamic system can be approximated by static NN, and applied it to the adaptive feedback linearization control problem for nonlinear system. The developed control system is simple in the construction and shows the adequate performance.

Keywords: Neural networks; Adaptive control; nonlinear control; Feedback linearization

1. Introduction

Neural networks since its advent, have been widely used because of its merits and specifications. Specifically, neural networks have the capability that can approximate the arbitrary continuous nonlinear mapping with enough accuracy, so can be effectively applied to automatic control engineering field such as nonlinear function approximation, system identification, modeling, adaptive control, fault tolerance control, parameter estimation and so on [5, 7, 12].

The most typical feature among them is a approach in which the unknown nonlinearity included in the control plant is approximated by one or more multi–layer feed–forward neural networks and the training policy of the NN is given so that the stability of the whole control system is guaranteed. In this approach no traditional algorithms such as BP algorithm are used, and the deflection signal that reflect the control system performance are directly used for training of NN instead of the explicit error signal in the output of NN, so that the control aim would be achieved. For example of such direct learning approach, in [1] Calise et al. present the adaptive output feedback control method for the nonlinear system based on the linear dynamic compensator and three–layer NN, and later in [2] improve it again using the linear observer. Lee et al. apply the similar adaptive control approach to the affine nonlinear system using two three–layer NNs in [4] and Vance et al. combine such adaptive control method with back–stepping approach so construct the NN control system for the discrete–time nonlinear affine plant using three single–hidden–layer NNs in [3]. For the nonlinear system with linearly connected input the application of such adaptive NN control is proposed by Zhang et al. in [8] and Meng et al. resolve the applying problem for tracking control of the non–affine nonlinear system in [6]. There are other many literatures deal with similar approach in order to realize the various control problems for the different plants such as cooperative tracking control of higher–order nonlinear system and nonlinear optimal control using adaptive dynamic programming etc.[10, 11].

In these approaches, although there exist some differences in detail learning algorithm or control scheme, but the main principles are mostly similar. Our research also treat the similar methodology but can make the control system structure more simple and improve the control performance. The important superiority of this research is that any error filter or dynamic compensator is not used in control system configuration. In previously proposed methods in order to guarantee the stability of tracking error the error filter with dimension equal to the relative degree of the plant is used and states of the filter is estimated by observer. In our method we only design state feedback controller and reference model without the error filter or observer.

The rest of this paper is parted as follows. In section 2, the construction of NN learning system is introduced and basic idea and principle is given. The stability analysis is shown in section 3. In section 4, the detail applications for nonlinear system control problem is given, simulation and experimental results are shown in section 5.

2. Problem statement

We consider a following SISO nonlinear system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bf(u) \\ y(t) &= cx(t) \end{aligned} \tag{1}$$

where

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + b\hat{f}(u) + L[y(t) - \hat{y}(t)] \\ \hat{y}(t) &= c\hat{x}(t) \end{aligned} \tag{2}$$

where $\hat{x}$, $\hat{y}$ are estimated values of the state and output of system respectively and $\hat{f}(\,)$ is a output of the three layer feed–forward NN described as follows:

$$\hat{f}(u) = W^T[\sigma(V^T u)] \tag{3}$$

where $W\in R^{nh}$, $V\in R^{nh}$ are weight vectors, and nh is the number of neurons in the hidden layer. And the activation function $\sigma(\ )$ is sigmoid function. For simplicity of description, we will fix weight vector V and only update vector W through the learning. A gain matrix of observer $L\in R^{n}$ is determined so that matrix $A-Lc^T$ is Hurwitz stable matrix. Therefore, dynamic of estimation error vector $e(t)=x(t)-\hat{x}(t)$ is represented as follows:

$$\dot{e}(t)=(A-Lc^T)e(t)+b[f(u)-\hat{f}(u)] \tag{4}$$

For ideal case that NN exactly approximate the unknown nonlinear function, estimation error would exponentially decrease to zero. Consequently the effect of free response due to initial deflection of state would gradually decrease and rest part of error corresponded to approximation error of NN would remain.

Therefore using output error signal between given system and observer system which includes the information for learning error of NN, we can train weight vector of NN. Let the learning algorithm is as follows similar to universal gradient descent algorithm.

$$\dot{\hat{W}}=\Gamma(\tilde{y}\phi(u)-k\hat{W}) \tag{5}$$

where $\hat{W}$ is estimation value of weight vector W, $\Gamma\in R^{nh\times nh}$ is positive symmetric matrix corresponds to the learning rate, $k>0$ is a design parameter and $\tilde{y}=y(t)-\hat{y}(t)$ is output error, and the notation $\phi(u)=\sigma(V^Tu)$ is used for simplicity. By this learning algorithm we can approximate the unknown nonlinear function by NN indirectly.

3. Stability analysis

The stability analysis of learning system shown in this section employs the Lyapunov analysis method, which has been widely used for demonstration of the stability of the NN control system.[9, 13]

Assumption 1.

The unknown nonlinear function $f(\ )$ in the dynamic system of Eq. (1) is bounded and the functional range of NN, $W_\phi(u)$ is dense over a considered compact domain $u\in D$ with respected of a finite set of bounded weights W.

This assumption is sufficiently permissible if the vector function $\phi(u)$ is selected as a basis over the domain of approximation and if the function $f(\ )$ is differentiable. [1, 2]

This means that there exist a NN such that approximate the nonlinear function in enough accuracy, i.e. nonlinear function $f(\ )$ can be represented as follows.

$$f(u)=W^T\sigma(V^Tu)+\varepsilon \tag{6}$$

According to the universal approximation property of NN we can get positive unknown constants w_m and ε_m holding $\|W\|\le w_m$ and $|\varepsilon|\le\varepsilon_m$, where $\|\cdot\|$ denotes Euclidean vector norm.[11]

Theorem 1.

For the NN learning system Eq. (1) and (2) under the assumption 1, tracking error and learning error signals are uniformly ultimately bounded.

Proof.

A Lyapunov−like function candidate is built as:

$$V=e^TPe+\tilde{W}^T\Gamma^{-1}\tilde{W} \tag{7}$$

where $\tilde{W}=W-\hat{W}$ and P is a positive definite matrix satisfy following inequality:

$$A_L^TP+PA_L+\gamma P^TP\le -Q \tag{8}$$

where $A_L=A-Lc^T$ and it is guaranteed by the properties of algebraic Riccati equation that there always exist a symmetric positive definite matrix P for arbitrary positive Q and $\gamma>0$ if A_L is Hurwichtz. Because both P and Γ are positive definite matrices, V is also positive definite.

By recalling (4), (5) and relation

$$\dot{e}=A_Le+b(\tilde{W}^T\phi+\varepsilon) \tag{9}$$

, the derivative of V is obtained as:

$$\begin{aligned}\dot{V}&=e^T(A_L^TP+PA_L)e+2e^TP(b\tilde{W}^T\phi(u)+b\varepsilon)\\&\quad-2\tilde{W}^T\Gamma^{-1}\dot{\hat{W}}\\&=e^T(A_L^TP+PA_L)e+2e^TPb\tilde{W}^T\phi(u)\\&\quad+2e^TPb\varepsilon-2\tilde{W}^T\tilde{y}\phi(u)+2k\tilde{W}^T\tilde{W}\end{aligned} \tag{10}$$

Using notation $\rho_1=\|b\|\varepsilon_m$, the relation $\|b\varepsilon\|\le\rho_1$ yields the inequality $2e^TPb\varepsilon\le e^TP^2e+\rho_1^2$. Thus Eq. (3.5) becomes

$$\begin{aligned}\dot{V}&\le e^T(A_L^TP+PA_L+P^TP)e+\rho_1^2+2e^TPb\tilde{W}^T\phi(u)\\&\quad-2e^Tc\tilde{W}^T\phi(u)+2k\tilde{W}^T\hat{W}\end{aligned} \tag{11}$$

Introducing the relation $b=b_0+b_1$, where $b_0=P^{-1}c$,

$$\begin{aligned}\dot{V}&\le e^T(A_L^TP+PA_L+P^TP)e+\rho_1^2+\\&\quad+2e^TP(b_0+b_1)\tilde{W}^T\phi(u)-2e^Tc\tilde{W}^T\phi(u)\ +2k\tilde{W}^T\hat{W}\\&=e^T(A_L^TP+PA_L+P^TP)e+\rho_1^2+2e^TPb_1\tilde{W}^T\phi(u)+2k\tilde{W}^T\hat{W}\end{aligned} \tag{12}$$

Using equality

$$2k\tilde{W}^T\hat{W}=k\|W\|^2-k\|\tilde{W}\|^2-k\|\hat{W}\|^2 \tag{13}$$

and inequality

$$2e^TPb_1\tilde{W}^T\phi(u)\le e^TP^TPe+\rho_2^2\|\tilde{W}\|^2 \tag{14}$$

where $\rho_2=\|b_1\|\phi_m$ with $\|\phi(u)\|\le\phi_m$, which is given by the boundedness of the sigmoid function.

$$\begin{aligned}\dot{V}&\le e^T(A_L^TP+PA_L+2P^TP)e+\rho_1^2+\rho_2^2\|\tilde{W}\|^2+\\&\quad+k\|W\|^2-k\|\tilde{W}\|^2-k\|\hat{W}\|^2\end{aligned} \tag{15}$$

Recalling Eq. (8), we get following result:

$$\dot{V} \le -e^T Qe - (k - \rho_2^2)\|\tilde{W}\|^2 - k\|\hat{W}\|^2 + \Lambda \tag{16}$$

where

$$\Lambda = \rho_1^2 + k\|W\|^2 \tag{17}$$

Hence if $k > \rho_2^2$, the derivative of V becomes negative in domain $\Omega_e = \left\{e \mid \|e\| \ge \sqrt{\Lambda / \lambda_{\min}(Q)}\right\}$ or $\Omega_w = \left\{\tilde{W} \mid \|\tilde{W}\| \ge \sqrt{\Lambda / (k - \rho_2^2)}\right\}$. According Lyapunov theorem, tracking error and learning error are uniformly ultimately bounded (UUB). At same time it is found that all the signals, including system output signal, are also UUB.

4. Design of the nonlinear control system

An observable and stabilizable single–input–single–output (SISO) nonlinear system commonly can be represented as follows:

$$\begin{aligned} \dot{x} &= f(x, u) \\ y &= g(x) \end{aligned} \tag{18}$$

where $x \in R^n$ is the state of the system, $u, y \in R$ are the input and output respectively, and $f(\ ,\ )$, $g(\ )$ are sufficiently smooth unknown nonlinear functions.

Assumption 2.

The system (18) satisfies the condition for output feedback linearization with relative degree r.

Following [3], then the system can be expressed in following form:

$$\begin{aligned} \dot{z}_i &= z_{i+1},\ i = 1, \cdots, r-1 \\ \dot{z}_r &= h(z, \zeta, u) \\ \dot{\zeta} &= f_0(z, \zeta) \\ y &= z_1 \end{aligned} \tag{19}$$

where $z = [y, \dot{y}, \cdots, y^{(r-1)}]$, $\zeta = [\zeta_1 \ \cdots \ \zeta_{n-r}]^T$ are the new state vector introduced again, $h(\ ,\ ) = L_f^{(r)} g$ is the Lie derivative function that obtained by transforming the system Eq. (18) and r is the relative degree.[6, 8]

Assumption 3.

The relative degree r is known, and the zero dynamics of the system (19) is exponentially stable on the considered compact set. Now we can apply the NN learning system for the identification or control of transformed system Eq. (19).

Considering the assumption of the feedback linearization condition, as there exist the inverse function of $h(\ ,\ )$, following feedback linearization control law is given. [2]

$$u = h^{-1}(v, z, \zeta) \tag{20}$$

where v is auxiliary input. Because the nonlinear function $h(\ ,\ )$ is unknown, we approximate it by appropriate invertible linear function $\hat{h}(y, u)$.

$$u = \hat{h}^{-1}(v, y) \tag{21}$$

Then the system dynamics is expressed as follows:

$$\begin{aligned} \dot{z} &= Az + b(v + \Delta) \\ y &= cz \end{aligned} \tag{22}$$

where Δ stands for the deviation term due to the approximation of unknown nonlinear function and can be denoted as follows:

$$\Delta(z, u, v) = h(z, \zeta, u) - \hat{h}(y, \hat{h}^{-1}(v, y)) \tag{23}$$

Moreover we let the auxiliary input to be a sum of the linearizing control input and the compensation term for canceling the deviation signal Δ as follows:

$$v = v_{lin} - v_{nn} \tag{24}$$

where v_{lin} is obtained from the linear controller designed to achieve the stability of closed loop system and v_{nn} is obtained as the output of a neural network to be trained to approximate the deviation signal.

If the NN compensate the deviation with enough precision, the system attached the feedback linearization scheme is expressed as a following linear system.

$$\begin{aligned} \dot{z} &= Az + bv_{lin} \\ y &= cz \end{aligned} \tag{25}$$

Therefore we construct the linear reference system in order to get the reference signals for the training of NN and state vector of feedback linearized system as follows:

$$\begin{aligned} \dot{\hat{z}} &= A\hat{z} + bv_{ref} \\ \hat{y} &= c\hat{z} \end{aligned} \tag{26}$$

where $\hat{z}$ is the state of reference system with same dimension of vector z and v_{ref} is the input signal of the reference system.

This reference model not only aims for estimating the system state but also plays a role that offers the reference signal for the NN to be trained. The input of the reference system v_{ref} is designed from linear state feedback control law as follows.

$$v_{ref} = K\hat{z} \tag{27}$$

The state feedback gain vector $K \in R^r$ is determined by well–known several methods such as LQ design method, pole placement method et al.

And the linearization input is composed as follows.

$$v_{lin} = v_{ref} - L\tilde{y} = K\hat{z} - L\tilde{y} \tag{28}$$

where $\tilde{y} = y - \hat{y} = ce$, $L \in R^r$ is determined such that matrix $A - Lc^T$ is stable.

Substituting Eq. (24) and Eq. (28) into Eq. (22) we can get following system equation:

$$\begin{aligned} \dot{z} &= Az + b(v_{ref} - L\tilde{y} - v_{nn} + \Delta) \\ y &= cz \end{aligned} \tag{29}$$

The state estimation error signal between the NN feedback linearization system and the linear model is described as following dynamic equation by subtracting Eq. (26) from Eq. (29)

$$\dot{e} = (A - Lc^T)e + b(\varDelta - v_{nn}) \tag{30}$$

where $e = z - \hat{z}$.

This error dynamic equation takes a same form of Eq. (21).

Hence we can train the NN with learning law Eq. (22) by using signal $\tilde{y}$ as well as nonlinear system identification method. Entire control system involving the reference system, the feedback linearization loop, NN and the state feedback controller is constructed as Fig. 1. From the theorem 1 and assumption 1–3, we can guarantee the stability of the closed control system if a matrix $A–bK$ is stable.

Remark: The state feedback in this control system has a same dynamic behaviour as the one proposed by Hovakimyan, because the state estimations are denoted as the high order derivatives of output.[1]

5. Simulation

In order to investigate the performance and specification of the proposed NN learning system and control system, we carry out the experiment and analysis for numerical sample model.

We consider a following nonlinear plant.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 - 0.2(x_1^2 + 1)x_2 - u \\ y &= x_1 \end{aligned}$$

The input of NN consists of following vector similar to above identification configuration.

$$i_{NN} = \begin{bmatrix} y_r(t) & \dot{y}_r(t) & u(t) \end{bmatrix}^T$$

The difference is that the output of reference system, not of plant was used. The 7 neurons of hidden layer are used and the state feedback gain is designed so that the poles of closed system are placed on [–0.6 –0.3] as follows.

$$K=[0.18\ 0.9]$$

Fig.2 plots the control responses of the system with periodic step reference signal and Fig.3 shows that with harmonic reference signal. In the first graph the solid curve is the output of the reference linear system and doted curve is the output of the plant. The second graph denotes the control input signal of the plant. From these curves we can see that the developed NN control system yields the good tracking performance. But the tracking process takes a bit of the phase delay feature.

6. Conclusions

In this paper we showed the learning algorithm and stability for the system in which the linear system combined with NN. Next based on it, introduced the methodology to realize the control for the nonlinear system. Finally through simulation results showed that proposed method can achieve the excellent performance. The proposed control method although displays the fine result for the plant with small nonlinearity such as example model used in simulation, but the control performance may be inferior for the plant with strong nonlinearity including the non–affine system. This can be solved by introducing the way that increases the number of neurons in the hidden layer of NN or that adjusts the learning rate vector in training process adaptively. And since the weights between hidden layer and output layer of NNs is trained in learning process, developing the learning rule for the weights between the input layer and hidden layer may improve the capacity of the learning system. Research for such sake is left as the future work.

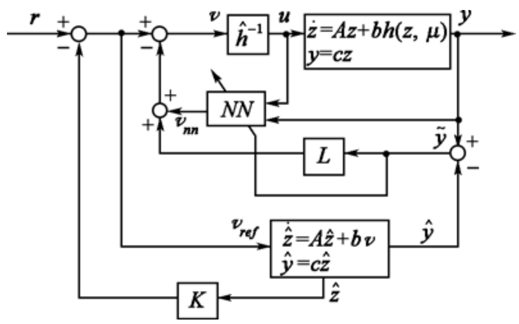

Figure 1. Configuration of the control system

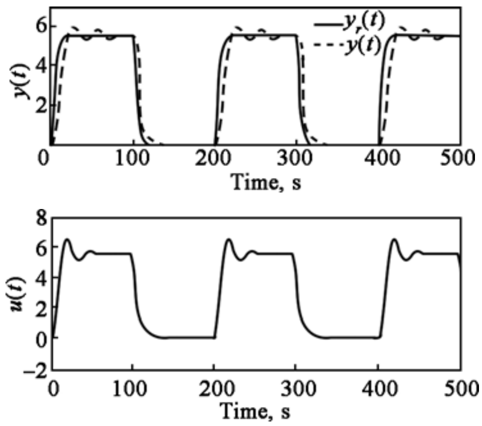

Figure 2. Simulation result of the control system with periodic step input signal

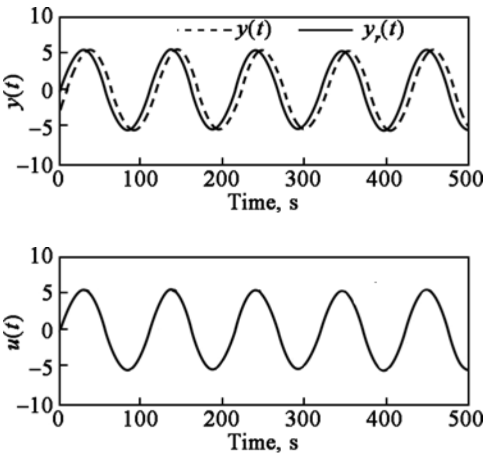

Figure 3. Simulation result of the control system with harmonic input signal

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