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Dynamic Model of Active Gas in Consideration of Effect of H_2O in Tens–MW TEA–CO_2 Laser

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Summary

We have developed an approach to modeling obtain the maximum output power for a high power TEA–CO_2 laser using active gases produced industrially which include considerable amount of impurities such as O_2 and H_2O molecules. In the paper we have discussed a dynamic modeling method and its verification results for a tens–MW TEA–CO_2 laser in exact consideration of the vibration energy interchange between CO_2, N_2, CO and H_2O molecules under enough cooling, the stimulated emission, the feature of optical cavity and the variation of electron density well–known.

Keywords: Vibrational temperature; H_2O molecule; Energy interchange;

1. Introduction

Concrete research on the energy interchange between particles during the discharge in the active gases produced industrially including impurities affords the possibility to cut down the cost of a high–power TEA–CO_2 laser.

As a result of lots of experimental researches on the effect of the impurities produced during the discharge and the main composition of active gases on the output power, the experimental data for theoretical study and simulation have been accumulated enough.[1–4] In order to find a dynamic model to ensure more accurate prediction of a TEA–CO_2 laser output power, we should study the energy interchange between the new compositions generated during the discharge and the main ones in the active gases based on the vibrational dynamics model. In addition we should take good care of the population change in the lasing levels, the stimulated emission, the effective and ineffective losses in the cavity and the electron density.[9] Researchers have developed rapidly methods of getting rid of the nitrogen oxides which have evil effect on lasing and presented models of the output power in consideration of five vibration temperature model including that of CO molecules. [6] When CO_2 and N_2 gases produced industrially are employed as active gases, it is reasonable to study the effect of the vibrational mode of H_2O molecules which are of considerable amount in the active gases on the output power. It has proved that H_2O molecules could play as much role of a collision molecule as that of He or Ar in the relaxation of CO_2 molecules from a level 010 to a level 000. [1, 7] Hence we claim that the energy interchange between H_2O molecules and others should be taken into account in designing a tens–MW TEA–CO_2 laser using active gases produced industrially without helium or argon. In the paper, we have presented a dynamic simulation and analysis of the laser in consideration of the temperatures of the active gases and lower vibration levels of CO_2, N_2, CO and H_2O molecules affecting on the output power of the laser.

2. Vibrational dynamics–based equation in consideration of effect of h_2o molecule in a level 010

With regarding that the energy difference between a level 010 of H_2O and a level 100 of CO_2 is not large, we set up the six vibrational dynamics–based equation classified by vibration mode. The first equation describes the variation of an energy density E_1 accumulated in a symmetry mode of a CO_2 molecule [9, 3, 11, 6, 7]:

$$\frac{dE_1}{dt} = -\frac{E_1 - E_1(T)}{\tau_{10}(T)} - \frac{E_1 - E_1(T,T_2)}{\tau_{12}(T,T_2)} - \frac{hv_1}{hv_6}\cdot\frac{E_6 - E_6(T,T_1)}{\tau_{61}(T,T_1)} + \frac{hv_1}{hv_3}\cdot\frac{E_3 - E_3(T,T_1,T_2)}{\tau_{312}(T,T_1,T_2)} + \frac{hv_1}{hv_5}\cdot\frac{E_5 - E_5(T,T_1,T_2)}{\tau_{512}(T,T_1,T_2)} + v_1 \Delta N_{31} W I_0 + P_{1E} \quad (1)$$

where the suffixes 0, 1, 2 and 3 stand for the ground state, the symmetric state, the deformation state, the asymmetric state of a CO_2 molecule, respectively, and 4, 5 and 6 the vibrational states of N_2, CO and a level 010 of a H_2O molecule, respectively. In fact, Eq.1 has been set up in consideration of the following processes:

$$CO_2(100)+M_t \rightarrow CO_2(000)+M_t$$

$$CO_2(100)+M_t \rightarrow CO_2(020)+M_t$$

$$CO_2(100)+H_2O(000) \rightarrow CO_2(000)+H_2O(010)$$

$$CO_2(001)+M_t \rightarrow CO_2(110)+M_t$$

$$CO(1)+CO_2(000) \rightarrow CO(0)+CO_2(110)$$

$$CO_2(000)+e \rightarrow CO_2(100)+e$$

where M_t stands for the kind of molecules (such as CO_2, N_2, H_2, H_2O, CO and O_2 molecules) colliding with a CO_2 molecule in a level 100 or 001. The sixth term in the right side of the Eq.1 displays that a CO_2 molecule transits from a level 001 to a level 100 emitting a photon at a wavelength of 10.6μm. In the term ΔN_{31} is the difference of the population between the two levels, I_{v0} the light intensity at the center frequency and W is given as

$$W = \frac{\lambda_1^2 g(v)}{8\pi v_1 \tau_{self}}$$

where λ_1 is the wavelength corresponding to $h\nu_1$, $g(\nu)$ the normalized spectral function, τ_{self} the lifetime of the spontaneous radiation.

The last term, P_{1E}, which is the energy received while CO_2 molecules transfer from a level 000 to a level 100 by collision with electrons in unit volume in unit time, is written as

$$P_{1E} = n_e(t) N_{CO_2} h\nu_1 \chi_1^e(T)$$

where $n_e(t)$ is the electron density, $\chi_1^e(T)$ the transition rate coefficient, f the degree of dissociation of the CO_2 molecules and N_{CO_2} the density of the CO_2 molecules.

Eventually, Eq.1 differs from the vibrational dynamics–based equations introduced in [9, 3, 11, 6, 7, 22] due to the reflection of the interaction between the symmetry mode of a CO_2 molecule and the deformation mode of a H_2O molecule.[7]

Similarly, the second and third equation state, respectively, the variation of the energy density E_2 of the deformation mode and an energy density E_3 of the asymmetric mode as follows

$$\begin{aligned}\frac{dE_2}{dt} = &-\frac{E_2 - E_2(T)}{\tau_{20}(T)} - \frac{h\nu_2}{h\nu_6}\cdot\frac{E_6 - E_6(T, T_2)}{\tau_{62}(T, T_2)} + \frac{h\nu_2}{h\nu_3}\cdot \\ &\cdot\frac{E_3 - E_3(T, T_1, T_2)}{\tau_{312}(T, T_1, T_2)} + \frac{E_1 - E_1(T, T_2)}{\tau_{12}(T, T_2)} + \frac{h\nu_2}{h\nu_5}\cdot \\ &\cdot\frac{E_5 - E_5(T, T_1, T_2)}{\tau_{512}(T, T_1, T_2)} + \frac{h\nu_2}{h\nu_3}\cdot\frac{E_3 - E_3(T, T_2)}{\tau_{32}(T, T_2)} + P_{2E}\end{aligned} \quad (2)$$

$$\begin{aligned}\frac{dE_3}{dt} = &-\frac{E_3 - E_3(T, T_1, T_2)}{\tau_3(T, T_1, T_2)} - \frac{E_3 - E_3(T, T_2)}{\tau_{32}(T, T_2)} + \frac{h\nu_3}{h\nu_4}\cdot \\ &\cdot\frac{E_4 - E_4(T, T_3)}{\tau_{43}(T, T_3)} + \frac{h\nu_3}{h\nu_5}\frac{E_5 - E_5(T, T_3)}{\tau_{53}(T, T_3)} + P_{3E} - \\ &- \nu_3 \Delta N_{31} W I_0(t)\end{aligned} \quad (3)$$

As shown in Eq.2, the influence of H_2O molecules has been also considered in Eq.2.

The fourth and fifth equation shows the variations of the energy densities of N_2 and CO molecules, respectively, as follows:

$$\begin{aligned}\frac{dE_4}{dt} = &-\frac{E_4 - E_4(T)}{\tau_{40}(T)} - \frac{E_4 - E_4(T, T_3)}{\tau_{43}(T, T_3)} + \frac{h\nu_4}{h\nu_5}\cdot \\ &\cdot\frac{E_5 - E_5(T, T_4)}{\tau_{54}(T, T_4)} + P_{4E}\end{aligned} \quad (4)$$

$$\begin{aligned}\frac{dE_5}{dt} = &-\frac{E_5 - E_5(T)}{\tau_{50}(T)} - \frac{E_5 - E_5(T, T_3)}{\tau_{53}(T, T_3)} + P_{5E} - \\ &-\frac{E_5 - E_5(T, T_1, T_2)}{\tau_{512}(T, T_1, T_2)} + \frac{h\nu_5}{h\nu_4}\frac{E_4 - E_4(T, T_5)}{\tau_{45}(T, T_5)}\end{aligned} \quad (5)$$

The last sixth equation explains the vibration of the energy density E_6 of H_2O molecule in a level $01^0 0$ as follows:

$$\begin{aligned}\frac{dE_6}{dt} = &P_{6E} + \frac{h\nu_1}{h\nu_6}\cdot\frac{E_6 - E_6(T, T_1)}{\tau_{61}(T, T_1)} + \frac{h\nu_2}{h\nu_6}\times \\ &\times\frac{E_6 - E_6(T, T_2)}{\tau_{62}(T, T_2)} - \frac{E_6 - E_6(T)}{\tau_{60}(T)}\end{aligned} \quad (6)$$

where the terms in the right side mean the energy interchange relation in the following processes, where we can see H_2O molecules play the same role as helium and argon atoms.

$$H_2O(000) + e \rightarrow H_2O(010) + e$$

$$H_2O(000) + CO_2(100) \rightarrow H_2O(010) + CO_2(100)$$

$$H_2O(000) + CO_2(020) \rightarrow H_2O(010) + CO_2(000)$$

$$H_2O(010) + M_t \rightarrow H_2O(000) + M_t$$

The above mentioned six equations are the basic ones of the output power model for a TEA–CO_2 laser.

3. Ourput power model base on the vibraitonal dynamics–based equations

The output power is derectly related to the population inversion. The population inversion ΔN is written as [15, 1, 7]

$$\Delta N = N_{001} \cdot P(J) - N_{100} \cdot P(J+1) \quad (7)$$

$$N_{001} = N_{CO_2} \exp(-h\nu_3 / kT_3) z \,, \quad N_{100} = N_{CO_2} \exp(-h\nu_1 / kT_1) z$$

$$z = \left[1 - \exp\left(\frac{h\nu_1}{kT_1}\right)\right] \cdot \left[1 - \exp\left(-\frac{h\nu_2}{kT_2}\right)\right]^2 \cdot \left[1 - \exp\left(-\frac{h\nu_3}{kT_3}\right)\right]$$

$$P(J) = (2hcB / kT) Q_J \exp[-hcBJ(J+1)/kT]$$

where B is the rotation constant of a CO_2 molecule, c the velocity of light and Q_J the degeneracy of a rotation state. As it is known, the increase of the radiation density by stimulated radiation per unit time is written as [4]

$$h\nu c^2 \Delta N I(\nu) \delta(\nu - \nu_0) g(\nu) d\nu / 8\pi h \nu^3 \tau_{self}$$

where c is the light velocity and $g(\nu)$ the normalized spectral line shape function. The delta function tells that the spectral line width of the laser emission is very narrow.

Integrating the above equation over the frequency, we obtain

$$\frac{\Delta N C^2 I_{\nu_0} g(\nu_0)}{8\pi \nu_0^2 \tau_{self}}$$

From this, the increase rate of the light intensity is written as follows

$$\left(\frac{dI_{\nu_0}}{dt}\right)_{\text{inc}} = \frac{\Delta N c^3 I_{\nu_0} g(\nu_0)}{8\pi \nu_0^2 \tau_{self}}$$

Meanwhile, all losses in the mirrors can be specified by a single parameter τ_{res} which is the lifetime of the cavity, and therefore we can write as follows

$$(dI / dt)_{los} = I / \tau_{res}$$

The minimum threshold of the population inversion is given at the maximum value of the spectral line shape function $g(\nu_0)$.

So the temporal change of the light intensity in the cavity can be written as follows [3]

$$\frac{dI_{\nu_0}}{dt} = -\frac{I_{\nu_0}}{\tau_{res}} + \frac{c^3 \Delta N I_{\nu_0}}{4\pi \nu_0^2 \Delta\nu \tau_{self}} = -\frac{I_{\nu_0}}{\tau_{res}} + c\nu_0 \Delta N \frac{\lambda_0^2}{4\pi^2 \nu_0 \tau_{self}} \cdot I_{\nu_0}$$

Now, the spontaneous radiation ought to be taken into account. From the general relation between the stimulated and spontaneous emission, the term related to the spontaneous emission is written as [3]

$$chvN_{0,\,0,\,1}P(J)\cdot G/\tau_{self}$$

where G states the share of the photons emitted spontaneously taking part in the laser beam. To be concrete, first, G represents the portion of the spontaneous radiation emitted as a small angle to the optical axis of the cavity, thereby, it being expressed as $\beta/(4\pi)$. Second, G represents the photons in the spectral interval dv which lies over the lasing threshold among those emitted spontaneously with a spectral linewidth Δv, thereby, it being displayed as follows:

$$g(v_0)dv \approx 2dv/\pi\Delta v$$

In our case, the magnitude of Δv is rather significant than the detailed expression of it.

As a result, the light intensity in the cavity can be determined by the following equation

$$\frac{dI_{v0}}{dt} = -\frac{I_{v0}}{\tau_{\text{res}}} + chv_0\left[\frac{\Delta NWI_{v0}}{h} + N_{0,0,1}P(J)S\right] \quad (8)$$

where

$$S = \beta dv/\left(2\pi^2\tau_{\text{self}}\Delta v\right)$$

In order to apply Eq.8 in the calculation of the laser output power, it is necessary to find the detailed expression of τ_{res}.

Assume that there is a stable cavity of a length L in the optic axis z, which is filled up with homogeneous active material of a gain α over a length l. Then we can write the following relation comes into being.

$$\alpha = -(1/2l)\ln[R(1-\delta)] \quad (9)$$

where R is the reflectivity of the output mirror and δ the sum of all the losses in the output mirror.

Therefore, the output power of the laser can be expressed as follows

$$P_{\text{out}} = A(1-R-\delta+\delta R)I_{v_0} \quad (10)$$

where A is the area of the cross section of the output mirror and I_{v0} the light intensity in it. It is impossible to calculate the flux density of the radiation energy $I(z)$ in static method but it's possible to do the average light intensity., i.e.

Considering Eq.8, we obtain

$$\langle I_{v_0}\rangle = -2[1-R(1-\delta)]I_{v_0}/\ln[R(1-\delta)] \quad (11)$$

and substituting I_{v0} into Eq.10

$$P_{\text{out}} = -\frac{A}{2}\ln[R(1-\delta)]\langle I_{v_0}\rangle\frac{1-R-\delta+\delta R}{1-R(1-\delta)} \quad (12)$$

Consequently, we have found an approach to calculating the laser output power.

$$\tau_{\text{res}} = -2l/C\ln[R(1-\delta)] \quad (13)$$

4. Simulation of output power and determination of amount of H_2O

In the second step, the solving we get the vibrational temperatures T_i(i=1 to 6). [7]

$$T_i = \frac{h\nu_i}{k\times\ln(1+N_i h\nu_i/E_i)}$$

where i=1 to 6 correspond to the vibrational modes of CO_2, N_2 and H_2O molecules. Then we compute from Eq. (7) to Eq. (13).

In the calculation we used the constants presented in [1 to 6] and the result in [2, 7] for the electron density $N_e(t)$.In third step, the optimization we find the amount of H_2O which gives the maximum power, and determine the basic parameters for a TEA–CO_2 laser with a peak power 100MW, a pulse width 0.1μs and an average power 50MW. Here the optimization variable chosen is the mixing proportion of the active gases produced industrially, which is one of the input parameters. We can easily modify this variable in practice.

We used the basic parameters which are data inputted in the calculation shown in **Table.1**.

Table 1. Basic parameters for design of 50MW TEA–CO_2 laser

Symbol	Magnitude	Symbol	Magnitude
hv_1	1 388cm^{-1}	C	2.998×10^{10}cm/s
hv_2	667cm^{-1}	k	1.38×10^{-6}erg/K
$\hbar v_3$	2 349cm^{-1}	J	20
hv_4	2 330cm^{-1}	B_{CO2}	0.4cm^{-1}
hv_5	2 150cm^{-1}		0.2 μs
hv_6	1 595cm^{-1}	p	760Torr
χ_1^2	5×10^{-9}cm^3/s	L	240cm
χ_2^2	3×10^{-9}cm^3/s	A	28.8cm^2
χ_3^2	8×10^{-9}cm^3/s	R	0.6
χ_4^2	2.3×10^{-}cm^3/s	s	0.13
χ_5^2	3×10^{-9}cm^3/s	f	0.2
χ_6^2	5×10^{-10}cm^3/s	F	0.8
h	6.625×10^{7}erg/s	CO_2: N_2: H_2O	10: 89.8 to 89: 0.2 to 1
λ	10.6μm	n_{e0}	10^{12}cm^{-3}

Fig. 1 shows the result of the simulation. The result tells that output power depends greatly on the amount of H_2O.

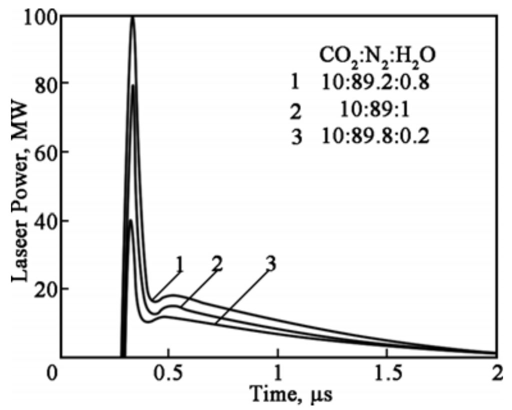

Figure 1. Calculation result for design base parameter of 50MW TEA–CO_2 laser.

5. Verification of simulation

First, we compared the simulation results to the experimental one showed in [6]. In the simulation, we used the same initial data as those in [6]. Fig. 2 shows the simulation result and Fig.3 the experimental result in [6]

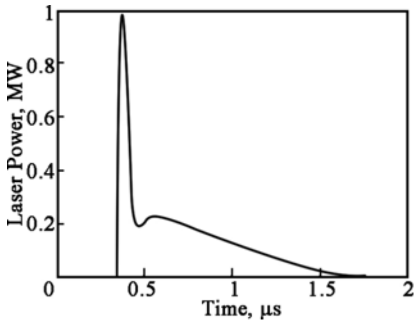

Figure 2. Simulation result

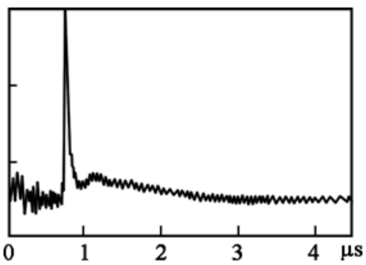

Figure 3. Experimental result in [6].

From comparison of Fig.2 with Fig.3, it is clear that both of results are almost equal in maximum pulse power and pulse width. Then we compared with our simulation with that of [7].

The initial data used in both of the simulations are the same. Because the active gases, which were not produced industrially, were used in [7], we took no account of the sixth vibrational temperature in the simulation.

Table 2. Initial data introduced in [7]

parameter	meaning	value	units
L	Length of resonator	116	cm
ι	Length of medium	100	cm
R	Reflection coefficient of output mirror	0.6	–
δ	Loss coefficient of resonator	0.1	–
f	Degree of dissociation of CO_2	0.2	–
A	Area of resonator mirror	16.8	cm^2
CO_2: N_2: He	Mixed ratio of active gas	18: 4: 78	
F	Space factor of active gas	0.43	

Fig. 4 shows the result of the simulation for theTEA–CO_2 laser output power and Fig.5 shows the result of [7].

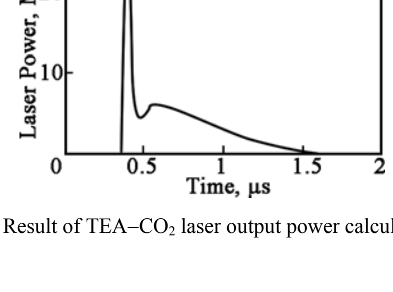

Figure 4. Result of TEA–CO_2 laser output power calculated by us.

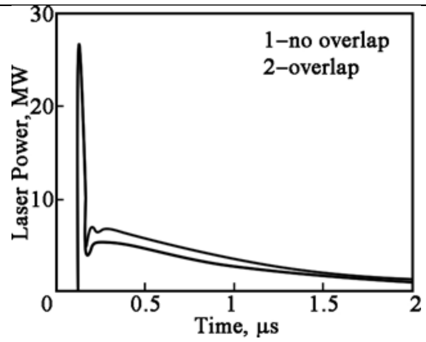

Figure 5. Result of TEA–CO_2 laser output power calculated in [7]

As shown Figure4 both of results are in good agreement in the waveform, the peak power of about 27.5MW and the pulse width of 0.1μs.

6. Conclusion

We have successfully developed an output power model taking into account the effect of H_2O molecules for a TEA–CO_2 laser using CO_2 and N_2 gases produced industrially. Because the model includes the temperature of the active gases as well as the six vibrational temperatures, it can be called a seven–temperature model. It has proved that the simulation result of the model was in good agreement with other results from references. This model and simulation method will give a methodology of designing the active gases of a tens–MW TEA–CO_2 laser using CO_2 and N_2 gases produced industrially. Besides the model can be further applied to developing a synthetic simulation to predict the parameters of the cavity, the sizes of the structures for pulse discharge under atmospheric pressure, the sizes of the components of the electric power and the parameters of the systems of gas circulation and cooling demanded for the total design of a TEA–CO_2 laser.

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